Port Reflection Coefficient Method for Solving Multi-Port Microwave Network Problems

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***Abstract*— The port reflection coefficient method (PRCM) is proposed for the treatment of multi-port microwave network systems. The theory is meaningful because in combination with other available numerical techniques, it can provide several possible ways for simplifying and solving complicated multi-port problems. The PRCM also suggests an approach for the measurement of scattering parameters, since it requires only the measurement of reflection coefficients at partial ports of the system. The efficiency and versatility of the method are verified through various numerical examples, including waveguide H-plane right angle bend, E-plane T-junctions, and multi-port power dividers. A special case of this method yields the well-known transverse resonance approach.**

I. Introduction

MULTI-port microwave network problems include, for example, two-port right angle bends, three-port T-junctions, and four-port cross junctions, etc. Other more complicated circuit components, like multiplexers and power dividers, can also be considered as multi-port microwave networks. Precise characterization of multi-port networks is very important because it is fundamental for the design of microwave circuits and systems. A few numerical and experimental techniques have been developed in the past for solving multi-port network problems. A recently proposed one is the Three Plane Mode-Matching Technique (TPMMT) [1], [2], which deals with three-port waveguide T-junctions.

When one of the authors tried to design rectangular coaxial line branch-type directional couplers years ago [3], [4], he developed an approach for treating two- and three-port networks. The work was extended recently by the authors, and the port reflection coefficient method (PRCM) for dealing with multi-port microwave networks is proposed in this paper. The main process of the PRCM can be described as follows. Among all the ports of the network, choose appropriately some at which we place short circuits, and the other ports are matched so that reflection coefficients at all ports of the modified configuration can be computed or measured in a convenient way. Repeat this process a certain number of timeswith different lengths of short circuits at the corresponding ports, we obtain several groups of reflection coefficients at all the ports (this depends on the number of ports of the system and the choice of ports at which short circuits are introduced. An example for the three-port case given in the next section will make this clear). After some mathematical formulation, the expected scattering parameters of the problem can be extracted from these reflection coefficients. The application of this process, with some modifications, to the design of rectangular coaxial line branch-type directional couplers resulted in great improvement of computation efficiency, since the original complicated three-dimensional problem was reduced to a two-dimensional cascaded discontinuity problem that could be solved by available numerical techniques [3], [4].

Actually, our PRCM originates from and is a generalization of the technique proposed in [5] for the treatment of two-port waveguide scattering problems by the finite-element method. In that technique [5], a short-circuit was used three times for obtaining three pairs of reflection coefficients, from which the scattering parameters were solved. Such a process, as pointed out by [5], was very similar to that of a standard experiment technique that was developed many years ago [6] for finding scattering parameters. We note that in the TPMMT [1], a short circuit was also utilized three times for obtaining reflection and transmission coefficients. For this reason, the TPMMT [1] and the PRCM of this paper are similar in this respect for the treatment of a three-port system.

Despite such a similarity, differences between the TPMMT and the PRCM prevail. The main features of the PRCM can be summarized as follows. First, in the PRCM, the expected scattering parameters are all extracted from the *reflection coefficients* obtained at all the ports, therefore what we try to calculate or measure are these reflection coefficients only. The method is thus named the "port reflection coefficient method." Next, the PRCM is suitable for the treatment of networks with more than three ports, taking into account interactions of higher-order modes among all the ports. Finally, the choice of ports at which short circuits are placed is flexible, therefore we have many possible ways to modify the configuration of the problem. Among these, we can choose one best suited for computation or measurement. A special case of the choices yields the transverse resonance approach [7], [8], and this will be made clear in the next section.

The significance of the PRCM lies in two aspects. First, in combination with other available numerical techniques, it can provide us several possible ways for simplifying and solving complicated muti-port circuit problems. Second, it also yields an approach for the measurement of scattering parameters, since it requires only the measurement of reflection coefficients at partial ports of the system. This is of particular importance because, as is well known, in many practical measurements standard high-quality loads are not available and short circuits are much more accurate and easier to make.

In the following text, first the theory of the PRCM is described in detail. Then, examples on waveguide H-plane right

Manuscript received January 22, 1994; revised May 15, 1994.
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IEEE Log Number 9407298.

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angle corner bends, E-plane T-junctions, and two- and three-series T-junction power dividers are provided. From these, applications of the PRCM, combined with other numerical techniques, for simplifying and solving complicated multi-port waveguide systems are demonstrated. Numerical results are compared with previously published data and experiment measurements, which proves the accuracy and versatility of the PRCM. Discussions on other techniques used for these examples are made, and finally, some conclusions concerning the use of the PRCM are given.

II. Port Reflection Coefficient Method

We assume that all the systems discussed below are reciprocal systems and begin our theory from a general two-port problem.

A. Two-Port Case

Fig. 1(a) shows a general two-port system. It can be considered as a discontinuity region connected with the outside through two transmission lines, which we denote as the port-1 and port-2, respectively. We close the port-2 by a short circuit for the later use, which we will explain at the end of this subsection. The length l_2 of the transmission line between the discontinuity region and the short circuit, as shown in Fig. 1(a), is taken to be sufficiently long so that the short circuit does not interact with the discontinuity region. Without loss of generality, we assume that only one operating mode can propagate at the port 1 and 2, respectively. Then, the initial system can be represented by the scattering matrix shown in Fig. 1(b), where a_1, a_2, b_1 and b_2, are incident and reflected waves at the port 1 and 2, respectively. The scattering expressions of this system are written as:

$$b_1 = S_{11}a_1 + S_{12}a_2 \qquad (1a)$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \qquad (1b)$$

If we denote the reflection coefficients at the port 1 and 2 by R_1 and R_2, respectively, and substitute the relations $b_1 = R_1a_1$ and $b_2 = R_2a_2$ into (1), we have:

$$(S_{11} - R_1)a_1 = -S_{12}a_2 \qquad (2a)$$

$$(S_{22} - R_2)a_2 = -S_{21}a_1 \qquad (2b)$$

Multiplying (2a) by (2b), we get

$$(S_{11} - R_1)(S_{22} - R_2) = S_{12}S_{21} \qquad (3)$$

In (3) we have three desired unknowns, S_{11}, S_{22} and $S_{12}S_{21}$. If we can obtain three pairs of reflection coefficients $(R_1^{(i)}, R_2^{(i)})$ $(i = 1, 2, 3)$ at the port 1 and 2, then we can solve S_{11} and S_{22} from (3) immediately (see (4a), shown at the bottom of this page).

$$S_{22} = \frac{R_1^{(1)}R_2^{(1)} - R_1^{(2)}R_2^{(2)} - (R_2^{(1)} - R_2^{(2)})S_{11}}{R_1^{(1)} - R_1^{(2)}} \qquad (4b)$$

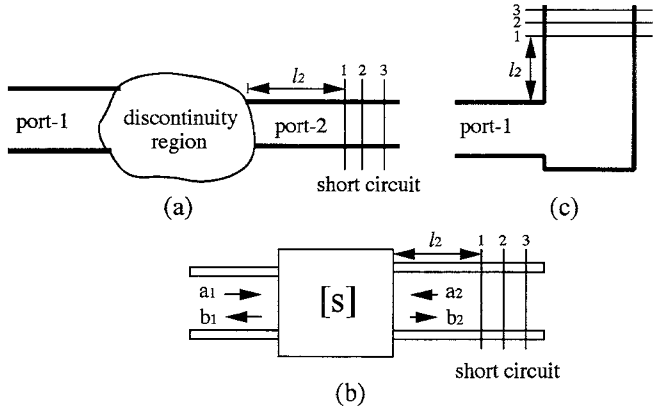

Fig. 1. (a) A general two-port system. (b) Scattering matrix representation of the two-port system. (c) Waveguide right angle corner bend.

For a reciprocal system, $S_{12} = S_{21}$, then from (3) we have

$$S_{21} = \pm\sqrt{(S_{11} - R_1^{(1)})(S_{22} - R_2^{(1)})} \qquad (4c)$$

For obtaining three pairs of reflection coefficients, $R_1^{(i)}$ and $R_2^{(i)}$ $(i = 1, 2, 3)$, at the port 1 and 2, there are of course many numerical and experimental techniques. One way is placing a short circuit at the port-2 and changing its position (*i.e.*, the length of $l_2^{(i)}$, as shown in Fig. 1) three times. The first advantage of this way lies in the fact that the reflection coefficient at the port-2 can be obtained in advance by:

$$R_2^{(i)} = -e^{j2\beta_2 l_2^{(i)}} \qquad (5)$$

Here β_2 is the phase constant of the operating mode at the port-2, and $l_2^{(i)}$ depends on our choice. What we need to find then is only the reflection coefficient $R_1^{(i)}$ at the port-1.

The second purpose of the short circuit is that, with the placement of it at the port-2, the configuration of the problem is modified so that the reflection coefficient $R_1^{(i)}$ at the port-1 may be calculated or measured in a more convenient way. Take the waveguide right angle corner bend shown in Fig. 1 (c) as an example. After the port-2 is terminated with a short circuit, the original structure is turned into a simple waveguide step-junction. Then, the reflection coefficient $R_1^{(i)}$ at the port-1 can be easily computed by, for example, the mode-matching method. Other examples include the waveguide T-junction and multi-port power dividers given in the next section, where with the aid of the short circuits, the modified configurations can all be treated as cascaded waveguide step-junctions.

It is obvious that the relations (1)–(4) are also valid if the reflection coefficients at the port 1 and 2 are obtained by other numerical or experimental approaches, such as the finite element method used in [5].

$$S_{11} = \frac{(R_1^{(1)}R_2^{(1)} - R_1^{(2)}R_2^{(2)})(R_1^{(1)} - R_1^{(3)}) - (R_1^{(1)}R_2^{(1)} - R_1^{(3)}R_2^{(3)})(R_1^{(1)} - R_1^{(2)})}{(R_2^{(1)} - R_2^{(2)})(R_1^{(1)} - R_1^{(3)}) - (R_2^{(1)} - R_2^{(3)})(R_1^{(1)} - R_1^{(2)})} \qquad (4a)$$

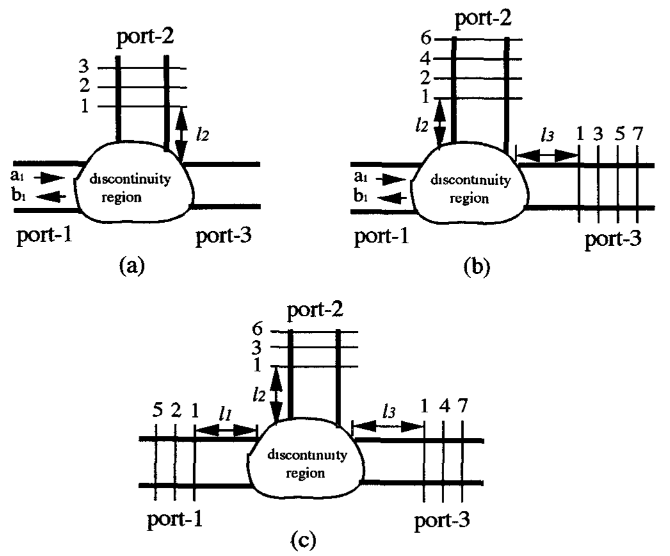

Fig. 2. Three approaches for solving a three-port network by the port reflection coefficient method. (a) Port-2 is terminated by a short-circuit. (b) Ports 2 and 3 are terminated by short-circuits. (c) All three ports are terminated by short-circuits. The lengths of short circuits on these ports are changed individually in an alternative sequence indicated by the numbers beside the short circuits.

B. Three-Port Case

Now we proceed with a three-port system shown in Fig. 2. As was stated in the introduction, since our choice of ports at which short circuits are placed is flexible, the three-port problem can be treated in the following three different approaches.

One-Port Short-Circuit Termination Approach: As shown in Fig. 2(a), we place a short circuit at *one* of the three ports, for example, the port-2 of the system. The scattering relations of this structure are:

$$b_1 = S_{11}a_1 + S_{21}a_2 + S_{31}a_3 \tag{6a}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 + S_{32}a_3 \tag{6b}$$

$$b_3 = S_{31}a_1 + S_{32}a_2 + S_{33}a_3 \tag{6c}$$

We assume that only one operating mode is incident at the port-1, and that port-3 is matched. Then, we have

$$b_1 = R_1 a_1, \qquad b_2 = R_2 a_2, \qquad a_3 = 0 \tag{7}$$

Substituting the above relations into (6a) and (6b), we have

$$(S_{11} - R_1)a_1 = -S_{21}a_2 \tag{8a}$$

$$(S_{22} - R_2)a_2 = -S_{21}a_1 \tag{8b}$$

which give

$$(S_{11} - R_1)(S_{22} - R_2) = S_{21}^2 \tag{9}$$

It is readily found that (8) and (9) are similar to (2) and (3) of the two-port system. Therefore, by following the procedure described above for the two-port case, *i.e.*, by changing the length $l_2^{(i)}$ of the short circuit three times and obtaining three pairs of reflection coefficients $(R_1^{(i)}, R_2^{(i)})$ $(i = 1, 2, 3)$, we can find S_{11}, S_{22} and S_{21} from (4) immediately.

To obtain solutions for S_{32} and S_{33}, we now assume that an operating mode is incident at the port-3 and that the port-1 is matched. Then,

$$b_3 = R_3 a_3, \qquad b_2 = R_2 a_2, \qquad a_1 = 0 \tag{10}$$

This time, from (6b), (6c), and (10) we have

$$(S_{22} - R_2)a_2 = -S_{32}a_3 \tag{11a}$$

$$(S_{33} - R_3)a_3 = -S_{32}a_2 \tag{11b}$$

and

$$(S_{22} - R_2)(S_{33} - R_3) = S_{32}^2 \tag{12}$$

As S_{22} is already known, we need now only two pairs of $(R_2^{(i)}, R_3^{(i)})$ $(i = 1, 2)$ to find S_{33} and S_{32} from (12):

$$S_{33} = \frac{R_2^{(1)}R_3^{(1)} - R_2^{(2)}R_3^{(2)} - (R_3^{(1)} - R_3^{(2)})S_{22}}{R_2^{(1)} - R_2^{(2)}} \tag{13a}$$

$$S_{32} = \pm\sqrt{(S_{22} - R_2^{(1)})(S_{33} - R_3^{(1)})} \tag{13b}$$

We summarize the above process from the point of view of numerical calculation and experimental measurement. In actual numerical calculations, when we change the length of the short circuit at the port-2 *three times*, we can usually get three groups of reflection coefficients $(R_1^{(i)}, R_2^{(i)}, R_3^{(i)})$ $(i = 1, 2, 3)$ *simultaneously*. Substituting the three pairs of $(R_1^{(i)}, R_2^{(i)})$ into (4), we get S_{11}, S_{22} and S_{21}. Substituting any two pairs of $(R_2^{(i)}, R_3^{(i)})$ into (13), we get S_{33} and S_{32}. In the case of experimental measurement, we first let the operating mode be incident at the port-1 and the port-3 be matched. By measuring the reflection coefficient $R_1^{(i)}$ at the port-1 *three times* with different lengths of the short circuit at the port-2, we obtain three pairs of $(R_1^{(i)}, R_2^{(i)})$ ($R_2^{(i)}$, determined by (5)). The scattering parameters S_{11}, S_{22} and S_{21} are then calculated from (4). Repeat this process *two times* with the signal source being linked to the port-3 and the port-1 being matched, to get two pairs of $(R_2^{(i)}, R_3^{(i)})$, from which, together with (13), we obtain S_{33} and S_{32}.

The parameter S_{31} is finally calculated by using the unitary property ($[\boldsymbol{S}]^T[\boldsymbol{S}]^* = [\boldsymbol{I}]$) of the scattering matrix [S], which gives

$$S_{31} = -(S_{11}S_{21}^* + S_{21}S_{22}^*)/S_{32}^* \tag{14}$$

Here, the mark * denotes the complex conjugate.

Two-Port Short-Circuit Termination Approach: This second approach places short circuits at *two* of the three ports, for example at the port-2 and 3 shown in Fig. 2(b), and calculating or measuring the reflection coefficient $R_1^{(i)}$ at the port-1 *seven times* with different lengths of short circuits at the port-2 and 3. In Fig. 2(b), we illustrate a possible choice for changing the lengths of short circuits at the port-2 and 3. As can be seen, the lengths of short circuits at the port-2 and 3 are varied three times, respectively, in an alternative sequence indicated by the numbers beside the short circuits. The reflection coefficients at the port-2 and 3 are determined by

$$R_2^{(i)} = -e^{j2\beta_2 l_2^{(i)}}, \qquad R_3^{(i)} = -e^{j2\beta_3 l_3^{(i)}} \tag{15}$$

Substituting the obtained seven groups of reflection coefficients $(R_1^{(i)}, R_2^{(i)}, R_3^{(i)})$ into (6), we have

$$\begin{vmatrix} S_{11} - R_1^{(i)} & S_{21} & S_{31} \\ S_{21} & S_{22} - R_2^{(i)} & S_{32} \\ S_{31} & S_{32} & S_{33} - R_3^{(i)} \end{vmatrix} = 0 \quad (16)$$

which is rewritten as

$$\begin{aligned} R_2^{(i)}R_3^{(i)}S_{11} &+ R_3^{(i)}R_1^{(i)}S_{22} + R_1^{(i)}R_2^{(i)}S_{33} \\ &+ R_1^{(i)}SS_{32} + R_2^{(i)}SS_{31} + R_3^{(i)}SS_{21} + SSS \\ &= R_1^{(i)}R_2^{(i)}R_3^{(i)}, \qquad i = 1, 2, \cdots, 7 \end{aligned} \quad (17)$$

where

$$SS_{32} = S_{32}^2 - S_{33}S_{22} \quad (18a)$$
$$SS_{31} = S_{31}^2 - S_{33}S_{11} \quad (18b)$$
$$SS_{21} = S_{21}^2 - S_{22}S_{11} \quad (18c)$$
$$\begin{aligned} SSS = S_{11}S_{22}S_{33} &- S_{11}S_{32}^2 \\ &- S_{22}S_{31}^2 - S_{33}S_{21}^2 + 2S_{21}S_{31}S_{32} \end{aligned} \quad (18d)$$

(17) constitutes linear simultaneous equations, from which all the variables, S_{11}, S_{22}, S_{33}, SS_{32}, SS_{31}, SS_{21} and SSS can be solved readily. Then, the desired S_{32}, S_{31} and S_{21} can be calculated from (18a)–(18c), which are rewritten in the following form:

$$S_{ij} = \pm\sqrt{SS_{ij} + S_{ii}S_{jj}}, \qquad i, j = 1, 2, 3, i > j \quad (19)$$

The choice of the sign $\pm$ in the above expression is arbitrary for any two of the parameters, S_{32}, S_{31} and S_{21}, but should be chosen for the remaining one so that the relation (18d) can be satisfied.

Comparing the procedures described above, we see that in the first one-short-circuit termination approach it is necessary to change the length of the short circuit *three* times for numerical calculation, but *five* times for experiment measurement. In the second two-short-circuit termination approach, it is necessary to change the lengths of the short circuits *seven* times for both the computation and the measurement. It is worth noting here that a measurement procedure similar to our second approach was also mentioned in [2] for a waveguide T-junction. However, *nine* times of measurements of reflection coefficients at the port-1 were required there.

All-Port Short-Circuit Termination Approach: In this approach, we place short circuits at all the three ports of the system. Then, the structure becomes a closed resonator, as shown by Fig. 2(c). Calculating or measuring *seven groups* of the resonant lengths $(l_1^{(i)}, l_2^{(i)}, l_3^{(i)})$ (In Fig. 2(c), the lengths of the short circuits at the three ports are varied two times, respectively, in an alternative sequence indicated by the numbers beside the short circuits), we obtain seven groups of reflection coefficients $(R_1^{(i)}, R_2^{(i)}, R_3^{(i)})$. Substituting these reflection coefficients into (17), we can solve all the expected scattering parameters, like we have done in the two-port-termination approach. We see such a process turns out to be similar to that of the transverse resonance technique proposed in [7] and [8]. The original scattering problem is thus turned into an eigenvalue problem in which we need to search for the eigenvalues and the resonant lengths $l_1^{(i)}, l_2^{(i)}$ and $l_3^{(i)}$ of the eigenvalue equation of the closed resonant structure. The computation time of this approach is, therefore, much longer than those of the first and the second approaches.

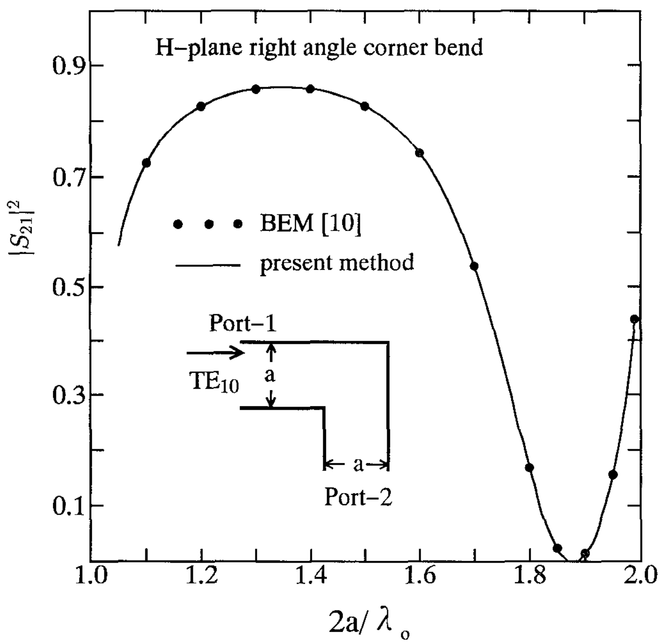

Fig. 3. Frequency dependence of the power transmission coefficient of a waveguide right angle corner bend.

C. Multi-Port Case

For a multi-port system, although the details of the solving process for the scattering parameters are different according to the number and the layout of ports of the problem, all the principles are the same as those described above for the three-port case: Among all the ports of the system, choose appropriately some at which short circuits are placed so that reflection coefficients at all the ports of the modified configuration can be computed or measured in an easier way. With some straightforward formulation similar to that shown above for the three-port case, the scattering parameters can be found from the obtained reflection coefficients. For the sake of brevity, we place mathematical formulations for a four-port system in the appendix and illustrate numerical results for a four- and a five-port waveguide power dividers in the following section.

III. Numerical Examples

First, we apply our theory to find the frequency-dependence transmission property of a waveguide H-plane right angle corner bend. As stated in the above section, when we place a short-circuit at one port of the structure the configuration becomes a simple waveguide step-junction (Fig. 1(c)). The reflection coefficient R_1 at the port-1 can be then easily obtained by using the mode-matching method [9]. The calculated result is shown in Fig. 3, and we see it agrees quite well with the result of [10] obtained by the boundary element method (BEM).

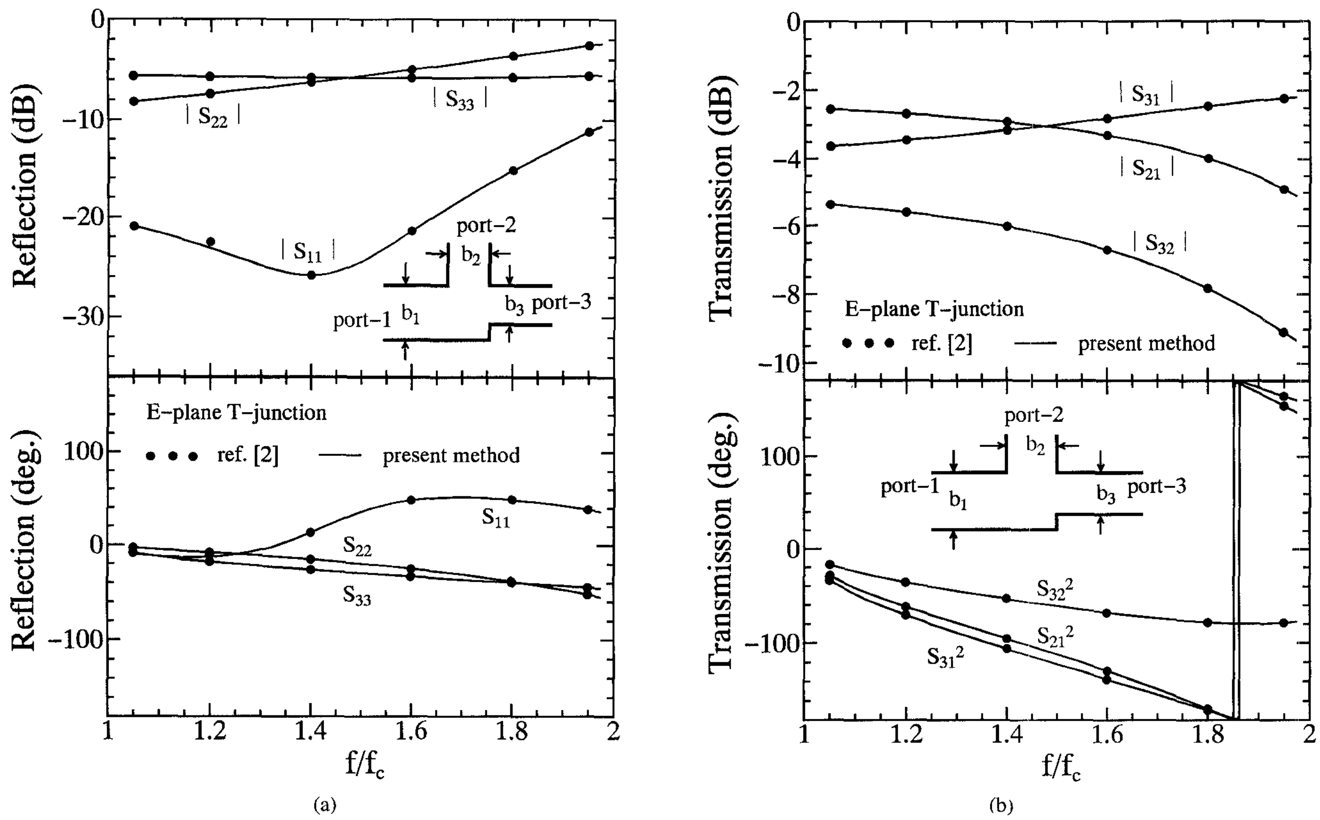

Fig. 4. Scattering characteristics of an E-plane T-junction. (a) Reflection coefficients. (b) Transmission coefficients. Waveguide dimensions: $a = 22.86$ mm, $b_1 = 10.16$ mm, $b_2 = 6.985$ mm, and $b_3 = 5.334$ mm.

To handle three-port systems, we described three approaches in the above section (Fig. 2). We tried all these three approaches on an asymmetrical waveguide E-plane T-junction, as shown by the inset of Fig. 4. The calculated results of the scattering parameters by these three approaches are coincident with negligible differences and are shown in Fig. 4. It is seen that both the amplitudes and the phases of our results agree well with those of [2] by the TPMMT. The computation times of the second approach (Fig. 2(b), with the port-2 and 3 being short-circuited), and the third approach (Fig. 2(c), with all three ports being short-circuited, i.e, the transverse resonance approach) are about 3 and 40 times, respectively, of that of the first approach (Fig. 2(a), with the arm port-2 being short circuited).

For a two-series T-junction waveguide power divider (Fig. 5, four-port problem), we investigate its scattering characteristics by placing short circuits at the port-2 and 3 so that the resulting structure can be viewed as consisting of cascaded waveguide step-junctions. Then, reflection coefficients at the port-1 and 4 can be solved by using the mode-matching method combined with the generalized scattering matrix technique [9], [11]. *Seven* groups of reflection coefficients are obtained by changing the lengths of short circuits at the port-2 and 3, alternatively (refer to the appendix and Fig. 8), and the desired scattering parameters are extracted from these reflection coefficients by using the mathematical formulations given in the appendix. Our results, as shown in Fig. 5, agree very well with those of [9], where a rigorous mode-matching analysis was also used. However, in the treatment of [9], the expansion of electromagnetic fields in the cavity regions was much more complicated.

The calculated characteristics of a 4-port waveguide E-plane power divider working at R-140 band are plotted in Fig. 6, and are compared with the measured data of [9]. The agreement is also quite well.

Finally, in Fig. 7, the scattering characteristics of a three-series T-junction waveguide power divider (five-port problem) are provided. Similar to the case of the two-series T-junction power divider, we place short circuits at the ports 2, 3, and 4, so that we only need to proceed with cascaded waveguide step-junctions and calculate reflections coefficients at the ports 1 and 5 *fifteen times* (there are fifteen unknowns in the 5×5 scattering matrix $[S]$ of the five-port structure) with different lengths of short circuits at the ports 2, 3, and 4. Since all the T-junctions are closely neighbored, higher-order modes among them interact with each other. Therefore, they cannot be handled individually. Our method treats the multi-port structure as a whole so that all the interactions among the higher-order modes are taken fully into account. We see that in Fig. 7 our results agree well with those of [9], and this validates again the present method for the treatment of multi-port problems.

IV. Conclusion

The port reflection coefficient method (PRCM) is proposed for the treatment of multi-port microwave network prob-

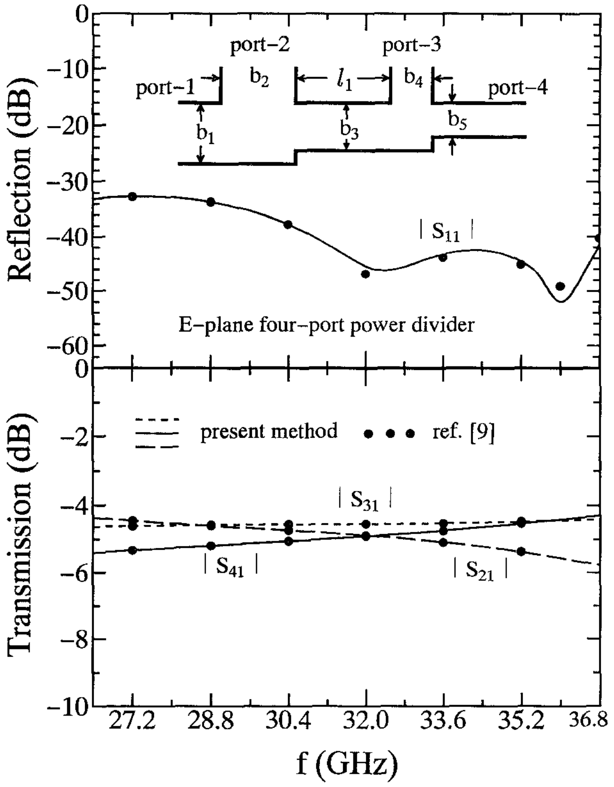

Fig. 5. Scattering characteristics of an E-plane two-series T-junction power divider. Waveguide dimensions (R320-band): a = 7.112 mm, b_1 = mm, b_2 = 1 50 3.556 mm, b_3 = 2.54 mm, b_4 = 1.61 mm, b_5 = 1.26 mm, and l_1 = 5.12 mm.

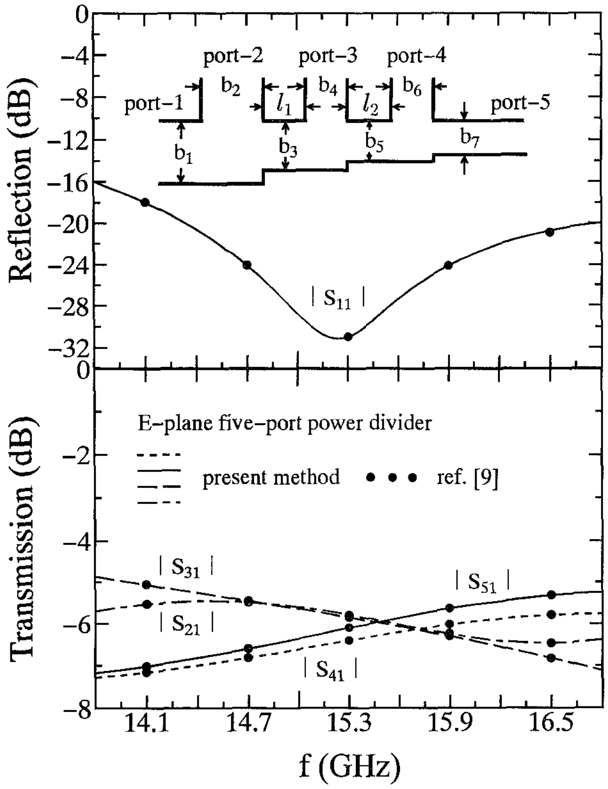

Fig. 7. Scattering characteristics of an E-plane three-series T-junction power divider. Waveguide dimensions (R140-band): a = 15.799 mm, b_1 = 7.8995 mm, b_2 = 2.65 mm, b_3 = 7.39 mm, b_4 = 2.67 mm, b_5 = 3.96 mm, b_6 = 3.67 mm, b_7 = 3.41 mm, l_1 = 7.95 mm, and l_2 = 7.94 mm.

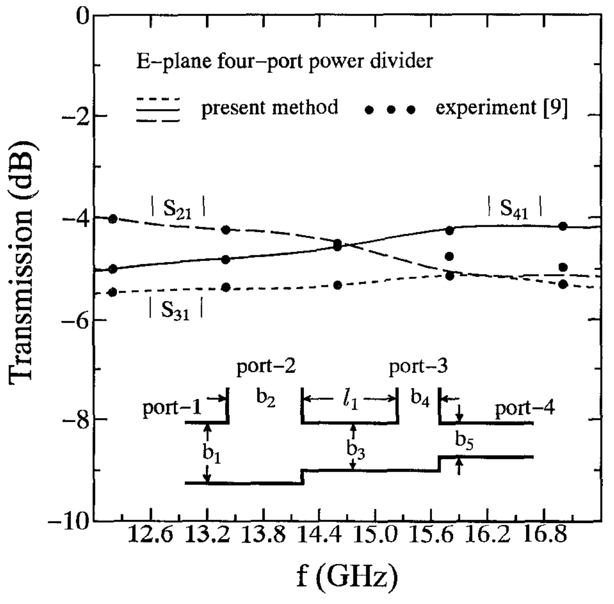

Fig. 6. Comparison between calculated and measured results of an R140-waveguide-band two-series T-junction power divider. Waveguide dimensions: a = 15.799 mm, b_1 = 7.8995 mm, b_2 = 3.95 mm, b_3 = 5.43 mm, b_4 = 2.83 mm, b_5 = 2.98 mm, and l_1 = 27.42 mm.

lems. Through various numerical examples, the efficiency and versatility of this method is verified. A special case of the method yields the well-known transverse resonance approach. Although all the numerical examples given in this paper are limited to the waveguide problems, the method itself is applicable to a wide variety of multi-port microwave network systems, provided that reflection coefficients at all the ports of the system are obtainable by available numerical or experimental techniques. Moreover, the method is also meaningful for the measurements of scattering parameters of a multi-port system, since it requires only the measurements of reflection coefficients at partial ports of the system.

APPENDIX

To solve the scattering characteristics of a four-port microwave system by the port reflection coefficient method, we have, as demonstrated for the three-port case in Section II, many possible ways for placing the short circuits. One way, as illustrated by Fig. 8, is placing short circuits at the ports 2 and 3 and changing the lengths of the two short circuits three times, respectively, in an alternative sequence indicated by the numbers beside the short circuits in Fig. 8. By means of numerical calculation or experimental measurement, we obtain *seven* pairs of reflection coefficients $(R_1^{(i)}, R_4^{(i)})$ $(i = 1, 2, \cdots, 7)$ at the ports 1 and 4, together with seven pairs of $(R_2^{(i)}, R_3^{(i)})$ calculated by using (15).

The scattering relations of the four-port network are:

$$b_1 = S_{11}a_1 + S_{21}a_2 + S_{31}a_3 + S_{41}a_4 \quad \text{(A1)}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 + S_{32}a_3 + S_{42}a_4 \quad \text{(A2)}$$

$$b_3 = S_{31}a_1 + S_{32}a_2 + S_{33}a_3 + S_{43}a_4 \quad \text{(A3)}$$

$$b_4 = S_{41}a_1 + S_{42}a_2 + S_{43}a_3 + S_{44}a_4 \quad \text{(A4)}$$

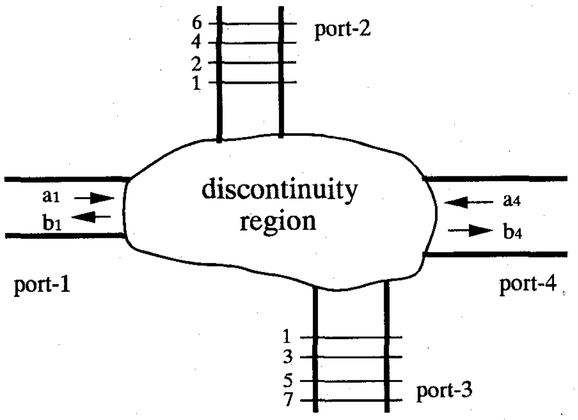

Fig. 8. An approach for placing short circuits on a 4-port network. The lengths of the short circuits at the ports 2 and 3 are changed, alternatively, in a sequence indicated by the numbers beside the short circuits.

Assuming

$$b_1 = R_1^{(i)} a_1,\ b_2 = R_2^{(i)} a_2,\ b_3 = R_3^{(i)} a_3, \text{ and } a_4 = 0 \quad \text{(A5)}$$

and substituting the above relations into (A1)–(A3), we have equations similar to (16) and (17). Using the obtained *seven* groups of $(R_1^{(i)}, R_2^{(i)}, R_3^{(i)})$ $(i = 1, 2, \cdots, 7)$ and solving the linear simultaneous equations (17), we can get S_{11}, S_{22}, S_{33}, S_{21}, S_{31} and S_{32}. Then, assuming

$$b_4 = R_4^{(i)} a_4,\ b_2 = R_2^{(i)} a_2,\ b_3 = R_3^{(i)} a_3, \text{ and } a_1 = 0 \quad \text{(A6)}$$

and substituting the above relations into (A2)–(A4), we again have equations similar to (16) and (17). This time we substitute *four* groups of $(R_2^{(i)}, R_3^{(i)}, R_4^{(i)})$ $(i = 1, 2, 3, 4)$ into the obtained linear simultaneous equations and get S_{44}, S_{43} and S_{42}. Finally, S_{41} is obtained by using the unitary relation of the scattering matrix [S]:

$$S_{41} = -(S_{11}S_{21}^* + S_{21}S_{22}^* + S_{31}S_{32}^*)/S_{42}^* \quad \text{(A7)}$$

ACKNOWLEDGEMENT

The authors thank Professor Shanjia Xu of the University of Science and Technology of China and Professor Naoto Kishi of the University of Electro-communications for their helpful discussions.

REFERENCES

[1] X. P. Liang, K. A. Zaki, and A. E. Atia, "A rigorous three plane mode-matching technique for characterizing waveguide T-junctions, and its application in multiplexer design," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-39, pp. 2138–2147, Dec. 1991.

[2] H. W. Yao, A. Abdelmonem, J. F. Liang, X. P. Liang, K. A. Zaki, and A. Martin, "Waveguide and ridge waveguide T-junctions for wide band applications," *IEEE MTT-S Int. Microwave Symp. Dig.*, pp. 601–604, 1993.

[3] S. Xu, W. Guo, X. Wu, Z. Li, Z. Ma, and W. Zhang, "Accurate analysis and experiment for rectangular coaxial line step discontinuity," in *Proc. 1992 Asia-Pacific Microwave Conf. (APMC'92)*, Adelaide, Australia, Aug., 1992, pp. 873–876.

[4] S. Xu, X. Wu, W. Guo, and Z. Li, " Scattering characteristics of rectangular coaxial line branching directional coupler," *IEEE Microwave and Guided Wave Lett.*, vol. 3, pp. 6–8, Jan. 1993.

[5] J. P. Webb, G. L. Maile, and R. L. Ferrari, "Finite-element solution of three-dimensional electromagnetic problems," *IEE Proc., Pt. H*, vol. 130, no. 2, pp. 153–159, Mar. 1983.

[6] M. Sucher and J. Fox, *Handbook of Microwave Measurements.* New York: Polytechnical Press, vol. 1, 1963.

[7] R. H. Jansen, "Hybrid mode analysis of end effects of planar microwave and millimeter-wave transmission lines," *Proc. Inst. Elec. Eng., pt. H*, vol. 128, pp. 77–86, 1981.

[8] R. Sorrentino and T. Itoh, "Transverse resonance analysis of finline discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1633–1638, Dec. 1984.

[9] F. Arndt, I. Ahrens, U. Papziner, U. Wiechmann, and R. Wilkeit, "Optimized E-plane T-junction series power dividers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 1052–1059, Nov. 1987.

[10] M. Koshiba and M. Suzuki, "Application of the boundary-element method to waveguide discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 301–307, Feb. 1986.

[11] Z. Ma, E. Yamashita and S. Xu, "Transverse scattering matrix formulation for a class of waveguide eigenvalue problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-41, pp. 1044–1051, June/July 1993.

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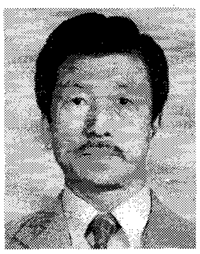

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